Kekule-distortion-induced exciton instability in graphene

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(Received 26 June 2008; published 19 September 2008)

Effects of a Kekule distortion on exciton instability in single-layer graphene are discussed. In the framework of quantum electrodynamics the mass of the electron generated dynamically is worked out using a Schwinger-Dyson equation. For homogeneous lattice distortion it is shown that the generated mass is independent of the amplitude of the lattice distortion at the one-loop approximation. Formation of excitons induced by the homogeneous Kekule distortion could appear only through direct dependence of the lattice distortion.

DOI: 10.1103/PhysRevB.78.115417 PACS number(s): 71.30.+h, 71.35.−y, 11.10.Kk, 81.05.Uw

I. INTRODUCTION

Recent experimental accessibility of graphene[1–3] has drawn much interest on this material that presents a wide variety of interesting properties.[4–8] Most of the properties of graphene arises from the peculiar energy spectrum near the so-called Dirac nodal points.[1,9]

Recent work on exciton instability in graphene monolayers is based on the Dirac Hamiltonian description.[10–12] The exciton gap is derived and solved through a self-consistent equation similar to the one appearing in the chiral symmetry-breaking phenomenon.[13] It was shown that an exciton can be formed under a strong long-ranged particle-hole interaction.[11,14] Exciton can also be formed in a single-layer graphene through the mechanism of magnetic catalysis of dynamical mass generation as pointed out in Ref. 15. This work showed that the magnetic catalysis can induce exciton condensation even for weak particle-hole coupling.[14] These results are obtained in the framework of quantum electrodynamics (QED) deduced from the linear energy spectrum of the graphene monolayer.

Exciton instability in graphene bilayer systems has been studied in the case of a short-ranged Coulomb interaction and a finite voltage difference between the layers.[16] Self-consistent exciton gap equations are derived in the framework of Hartree-Fock approximation and it is shown that a critical strength of the Coulomb interaction exists for the formation of excitons. The critical strength depends on the amount of voltage difference between the layers and on the interlayer hopping parameter. The voltage difference drives a gap in the energy spectrum of the graphene bilayer[17,18] and combined to a strong Coulomb interaction leads to an exciton instability.

Similarly a gap can open in the energy spectrum of graphene monolayers by means of a lattice distortion such as the so-called Kekule distortion.[19–21] A weakly screened Coulomb interaction and a gap in the linear energy spectrum of the graphene monolayer are favorable elements for the formation of excitons.

We focus our attention here on the consequences of a Kekule distortion on dynamical mass generation for electrons in the graphene monolayer. Indeed quasiparticle in two-dimensional (2D) systems can acquire mass through dynamical interaction between electrons and holes leading to an increase in the energy gap.[13,22,23] We address the question: can a Kekule distortion affect the dynamical mass generation mechanism in such a way that the resulting energy gap favors the formation of excitons? We show that in the specific case of a homogeneous Kekule distortion and at the one-loop approximation the dynamical mass generation is unaffected by the lattice distortion. We show that the gap in the energy spectrum is a sum of two independent contributions: one induced by the homogeneous Kekule distortion and one induced by dynamical mass generation. Consequently exciton instability can only be formed through direct dependence of the amplitude of the lattice distortion and the Kekule-independent dynamical mass of the electrons. There are no amplification effects of the homogeneous Kekule distortion by the mechanism of dynamical mass generation.

The outline of the paper is as follows: In Sec. II we recall the construction of the quantum electrodynamic action of the graphene monolayer in presence of a Kekule distortion. In Sec. III we present the calculation of the dynamical mass term using the Schwinger-Dyson equation of the electron. Section IV summarizes and discusses the present work.

II. QED$_3$ ACTION OF THE GRAPHENE MONOLAYER WITH A KEKULE DISTORTION

A graphene monolayer is a honeycomb array of atoms of carbon as depicted in Fig. 1. In a monolayer the electrons can hop between nearest-neighbor carbon atoms through π orbitals with energy t. The unit cell of the graphene monolayer is composed of two types of carbon atoms and we denote them by A and B. The whole lattice is divided into two sublattices, the Λ_A type and the Λ_B type. The Hamiltonian describing the graphene monolayer reads

$$H_0 = -t \sum_{r\in\Lambda_A} \sum_{\alpha=1}^{3} a_r^\dagger b_{r+r_\alpha} + \text{H.c.}, \tag{1}$$

where a and b stand for the π-electron creation and annihilation fermionic operator on the atoms of types A and B, respectively. The vector r_α connects the two sublattices Λ_A and Λ_B and is given by $r_\alpha = a e_\alpha$ with a the lattice parameter and $e_1=(1,0)$, $e_2=(-1/2,\sqrt{3}/2)$, and $e_3=(-1/2,-\sqrt{3}/2)$ are unit vectors. In the following the lattice parameter a will be set equal to one and plays the role of the unit of length.

The diagonalization of the Hamiltonian (1) leads to the kinetic energy $\varepsilon_k = t\left|\Sigma_{\alpha=1}^{3} e^{ik.e_\alpha}\right|$, where e_α's are the nearest-

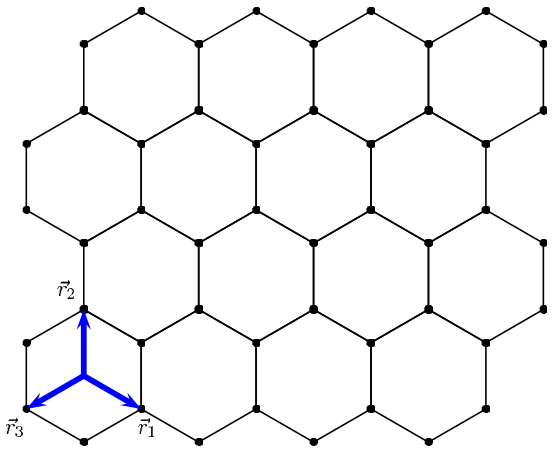

FIG. 1. (Color online) Representation of the graphene monolayer with the nearest-neighbor vectors r_α connecting the two sublattices Λ_A and Λ_B.

neighbor vectors of the graphene monolayer. The kinetic energy vanishes at the two independent nodal points $K_{+(-)}$, which are chosen as $K_+=(0,\frac{4\pi}{3\sqrt{3}})$ and $K_-=-K_+$ in the Brillouin zone. At low energy the bare Hamiltonian H_0 of the π electron can be rewritten by considering the energetic contributions around the nodal points and reads[9]

$$H_0=-\left\{\sum_r v_F u_a^\dagger(r)(2\partial_{\bar{z}})u_b(r)+\sum_r v_F v_a^\dagger(r)(-2\partial_z)v_b(r)+\text{H.c.}\right\},$$

where we made use of the notations in the complex plan $z=x+iy$ and $\partial_z=\frac{1}{2}(\partial_x-i\partial_y)$. The velocity $v_F=\frac{3}{2}ta/\hbar$, where we set $\hbar=a=1$ (the kinetic energy becomes $\varepsilon_p=\hbar v_F|p|$). The amplitudes $u_a(b)$ and $v_a(b)$ are smooth functional operators and are connected to the creation and annihilation operators a and b through the Fourier components $u_a(p)=a_{p+K_+}, u_b(p)=b_{p+K_+}$ and $v_a(p)=a_{p+K_-}, v_b(p)=b_{p+K_-}$, where p is the wave vector. The Hamiltonian H_0 describes the hopping of the π electrons in an undistorted lattice.

A lattice distortion can be modeled by a variation in the hopping parameter $\delta t_{r,\alpha}$ that depends on the position in the lattice $\vec{r}$ and on the direction e_α. The corresponding Hamiltonian for the graphene monolayer reads

$$H_K=-\sum_{r\in\Lambda_A}\sum_{\alpha=1}^{3}\delta t_{r,\alpha}a_r^\dagger b_{r+b_\alpha}+\text{H.c.} \tag{2}$$

In the following we consider the Kekule distortion for which the bonds between the carbon atoms are arranged similar to the benzene molecule.[19] The lattice is then pictured by an alternation of long and short bonds between the carbon atoms of types A and B.[24] The Kekule distortion is modeled by the hopping parameter $\delta t_{r,\alpha}$ and reads $\delta t_{r,\alpha}=\frac{1}{3}[\Delta(r)e^{iK_+r_\alpha}e^{iG.r}+\bar{\Delta}(r)e^{iK_-r_\alpha}e^{-iG.r}]$, where $\Delta(r)$ stands for the amplitude of the Kekule distortion and $K_{+(-)}$ are the nodal points in the Brillouin zone. The vector $G=K_+-K_-$ connects the two independent Dirac cones located at the nodal points $K_{+(-)}$. Focusing on the low-energy contributions of the Kekule distortion around the nodal points the Hamiltonian H_K is given by

$$H_K=-\sum_{r\in\Lambda_A}[\Delta(r)u_a^\dagger(r)v_b(r)+\bar{\Delta}(r)v_a^\dagger(r)u_b(r)]+\text{H.c.}$$

The Hamiltonian describing the π electrons in the graphene single layer with a Kekule distortion is then the sum of the bare and Kekule Hamiltonians $H=H_0+H_K$. Using the fermionic spinor $\psi^\dagger=[u_b(r)u_a(r)v_a(r)v_b(r)]^\dagger$ the Hamiltonian H can be rewritten in the quantum electrodynamic framework and reads

$$H=\int d^2\vec{r}\bar{\psi}(r)[v_F\gamma_k\partial_k+\tilde{\tilde{\Delta}}(r)]\psi(r),$$

where the gamma matrices are defined by $\gamma_0=\tau_3\otimes\tau_3$, $\gamma_1=v_F\tau_1\otimes\tau_3$ and $\gamma_2=v_F\tau_2\otimes\tau_3$, and $\tau_{\{0,1,2\}}$ are the Pauli matrices. The element $\tilde{\tilde{\Delta}}(r)$ is 4×4 matrices related to the Kekule-distortion amplitude $\Delta(r)$ by the relation

$$\tilde{\tilde{\Delta}}(r)=\begin{pmatrix}0 & \Delta(r)\tau_3\\ -\bar{\Delta}(r)\tau_3 & 0\end{pmatrix}$$

. The aim of the present work is to characterize the behavior of the dynamical mass generation for graphene monolayer in presence of a homogeneous Kekule distortion; we reduce our study to the case $\Delta(r)=\Delta_0$. Finally for a system at temperature $T=1/\beta$ the action of the graphene monolayer is given by

$$S_{\text{el}}=\int_0^\beta d\tau\int d^2\vec{r}\bar{\psi}(r,\tau)[\gamma^\mu\partial_\mu+\tilde{\tilde{\Delta}}_0]\psi(r,\tau). \tag{3}$$

The presence of the electromagnetic field surrounding the graphene monolayer is also to be considered. Indeed the graphene monolayer is embedded in an electromagnetic field that spans the whole three-dimensional (3D) space. However the electrons are confined in the 2D space delimited by the carbon atoms. Hence the density of charge and current verify $\rho(x,y,z)=\delta(z)\rho_{2D}(x,y)$ and $\vec{J}(x,y,z)=\delta(z)\vec{J}_{2D}(x,y)$. The electromagnetic field vector $\vec{a}$ and scalar ϕ potentials are related to the Green functions of the Laplacian $2\sqrt{-\vec{\nabla}^2}$ rather than $\vec{\nabla}^2$ for the three-dimensional electromagnetic field.[22,25] To describe the electromagnetic field embedding the graphene monolayer it is convenient to use the following Euclidean QED_3 action:

$$S_{e.m.}=-\int d^3x\frac{1}{2\sqrt{-\partial^2}}f^{\mu\nu}f_{\mu\nu}, \tag{4}$$

where $f_{\mu\nu}=\partial_\mu a_\nu-\partial_\nu a_\mu$ is the electromagnetic field tensor derived from the three-dimensional vector potential a_μ. The Fourier transform of action (4) leads to the bare photon propagator $\Delta_{\mu\nu}^{(0)}=(\delta_{\mu\nu}-q_\mu q_\nu)/(2|q|)$ in Euclidean space. For finite temperature the imaginary-time component q_0 of the $(2+1)$-dimensional wave vector $q=(q_0,q_1,q_2)$ is given by the bosonic Matsubara frequency $q_0=2\pi n/\beta$ (n is an integer and takes its values in the range $]-\infty,\infty[$).

Finally the full quantum electrodynamic action describing the graphene monolayer with a homogeneous Kekule distor-

tion and embedded in a 3D electromagnetic field reads $S = S_{e.m.} + S_{\mathrm{el}}$,

$$S = \int d^3x \frac{-1}{2\sqrt{-\partial^2}} f_{\mu\nu} f^{\mu\nu} + \int d^3x \bar{\psi}[\gamma^\mu(\partial_\mu - iga_\mu) + \tilde{\tilde{\Delta}}_0]\psi. \tag{5}$$

It is known that 2D systems described by a quantum electrodynamic action such as Eq. (5) experience a dynamical mass generation for the fermionic field ψ in interaction with a $U(1)$ gauge field. Appelquist and co-workers[13,23] showed that at zero temperature the originally massless fermions can acquire a dynamically generated mass when the number N of fermion flavors is lower than the critical value $N_c = 32/\pi^2$. Later Maris[26] confirmed the existence of a critical value $N_c \simeq 3.3$ below which the dynamical mass can be generated. Since we consider only spin-1/2 systems, $N=2$ and hence $N < N_c$. At finite temperature Dorey and Mavromatos[22] and Lee[27] showed that the dynamically generated mass vanishes at a temperature T larger than the critical one T_c. More recent works have been performed on dynamical mass generation in graphene monolayer.[10,11] However a question now arises about the effects of a lattice distortion such as the Kekule distortion. How does the mass generated dynamically in presence of a Kekule distortion behave? We concentrate on the effects of a homogeneous Kekule distortion and for small temperature $T \to 0$.

III. DYNAMICAL MASS GENERATION

A. The photon propagator at finite temperature

Integrating over the fermion fields ψ leads to a pure gauge Lagrangian $\mathcal{L}_a = \frac{1}{2} a_\mu \Delta^{-1}_{\mu\nu} a_\nu$, where $\Delta_{\mu\nu}$ is the dressed photon propagator from which we shall extract an effective interaction potential between two fermions and derive the dynamical mass of the fermions.

The finite temperature photon propagator in Euclidean space verifies the Dyson equation[28,29]

$$\Delta^{-1}_{\mu\nu} = \Delta^{(0)^{-1}}_{\mu\nu} + \Pi_{\mu\nu},$$

where the bare photon propagator $\Delta^{(0)}_{\mu\nu}$ is derived from the action of the bare electromagnetic field, Eq. (4), and the polarization function is obtained from the integration over the fermionic field ψ in the action (5).

For the computation of the dynamically generated mass it is enough to consider the static temporal dressed photon propagator component $\Delta_{00}(q^0=0,\vec{q})$ for which the bare photon propagator component reads $\Delta^{(0)}_{00}(q^0=0,\vec{q}) = 1/2|q|$. The detailed calculation of the static temporal component of the polarization function $\Pi_{00}(q^0=0,\vec{q})$ is given in Appendix and reads

$$\Pi^{00}(q^0=0,\vec{q}) = \int_0^1 dx \left(\frac{\alpha}{2\pi v_F^2 \beta}\right) \left\{ \ln[2\cosh(\pi\Theta_q)] - \left(\frac{\beta|\Delta_0|}{2}\right)^2 \frac{\tanh(\pi\Theta_q)}{(\pi\Theta_q)} \right\}, \tag{6}$$

where β is the inverse temperature and $\Theta_q = (\frac{\beta}{2\pi})\sqrt{x(1-x)v_F^2\vec{q}^2 + |\Delta_0|^2}$. The coupling parameter α is related to the number of fermion flavor $N=2$ and to the electron charge g, $\alpha = 4g^2N$.

For very small temperature $T \to 0$ the polarization function (6) becomes

$$\Pi^{00}(q^0=0,\vec{q}) = \left(\frac{\alpha}{8\pi v_F^2}\right)\left[|\Delta_0| + \frac{(v_F q)^2 - 4|\Delta_0|^2}{2v_F|q|} \arctan\left(\frac{v_F q}{2|\Delta_0|}\right)\right].$$

The process of dynamical mass generation is dominated by mechanisms at large wave vectors $\vec{q}$. Indeed for $q \gg 2|\Delta_0|/v_F$ the polarization function is asymptotically equal to $\frac{\alpha}{16\pi v_F}|q|$, which confirms the fact that the dynamics between electrons and holes are dominated by large wave vectors.[14]

B. The electron self-energy

We now derive the electron self-energy, which is also the dynamical mass. The Schwinger-Dyson equation for the electron propagator at finite temperature reads

$$G^{-1}(k) = G^{(0)^{-1}}(k) - \frac{g}{\beta}\sum_{\tilde{\omega}_{F,n}} \int \frac{d^2\vec{P}}{(2\pi)^2} \gamma_\mu G(p) \Delta_{\mu\nu}(k-p)\Gamma_\nu, \tag{7}$$

where $p = (p_0 = \tilde{\omega}_{F,n}, \vec{P})$, G is the dressed electron propagator, Γ_ν the electron-photon vertex, which will be approximated here by its bare value $g\gamma_\nu$, and $\Delta_{\mu\nu}$ is the dressed photon propagator. The second term in Eq. (7) is the fermion self-energy Σ ($G^{-1} = G^{(0)^{-1}} - \Sigma$). Performing the trace over the γ matrices and working out the sum over the fermionic Matsubara frequencies $\tilde{\omega}_{F,n}$ in Eq. (7) lead to a self-consistent equation for the self-energy of the form

$$\Sigma(\vec{k}) = g^2 \int \frac{d^2\vec{p}}{(2\pi)^2} \Delta_{00}(0,\vec{k}-\vec{p}) \frac{\Sigma(\vec{p})}{2\varepsilon_{\vec{p}}} \tanh\left(\frac{\beta\varepsilon_{\vec{p}}}{2}\right), \tag{8}$$

where $\varepsilon_{\vec{p}} = \sqrt{\varepsilon^{(0)2}_{\vec{p}} + \Sigma(p)^2}$ and $\varepsilon^{(0)2}_{\vec{p}} = v_F^2\vec{p}^2 + |\Delta_0|^2$ are, respectively, the energy spectrum of the graphene monolayer with and without corrections of the one-loop approximation near the nodal points $K_{+(-)}$. In the limit $q \gg 2|\Delta_0|/v_F$ the dressed photon propagator is given by $\Delta_{00}(0,\vec{q}) = \{[2 + \alpha/(16\pi v_F)]|q|\}^{-1}$. The angular integration in Eq. (8) is achieved using the approximation $f(|\vec{p}-\vec{k}|) = \theta(p-k)f(|p|) + \theta(k-p)f(|k|)$, where f is a function that depends on the absolute value of its arguments.[12,14] Hence the self-consistent Eq. (8) reduces to

$$\Sigma(k) = C\left\{ \int_0^k dp \frac{p\Sigma(p)}{k\sqrt{p^2 + [\tilde{\Delta}(p)/v_F]^2}} + \int_k^\Lambda dp \frac{\Sigma(p)}{\sqrt{p^2 + [\tilde{\Delta}(p)/v_F]^2}} \right\}, \tag{9}$$

where we introduced the constant $C = 4g^2/(32\pi v_F + \alpha)$, the

function $\tilde{\Delta}(p)=\sqrt{|\Delta_0|^2+\Sigma(p)^2}$, and the ultraviolet cutoff Λ. The electron self-energy is assumed to be a small correction to the electron propagator. It follows that for $k \gg |\Delta_0|/v_F$ the denominator of the second term in Eq. (9) is approximately equal to p. In the first term of Eq. (9) the function $\tilde{\Delta}(p)$ plays the role of a cutoff for $p \to 0$ and Eq. (9) can be rewritten as $\Sigma(k)=C[\int_{\tilde{\Delta}(p=0)/v_F}^{k} dp \frac{\Sigma(p)}{k}+\int_k^{\Lambda} dp \frac{\Sigma(p)}{p}]$. A derivation of this equation up to the second order with respect to the wave vector k of the electron self-energy leads to the differential equation

$$k^2\Sigma''(k)+2k\Sigma'(k)+C\Sigma(k)=0, \tag{10}$$

for which the infrared and ultraviolet boundary conditions are given, respectively, by

$$k^2\Sigma'(k)|_{k=\tilde{\Delta}/v_F}=0, \tag{11}$$

and

$$[k\Sigma'(k)+\Sigma(k)]|_{k=\Lambda}=0. \tag{12}$$

The differential Eq. (10) admits a solution that verifies also the boundary conditions (11) and (12) and reads[12,14,23]

$$\Sigma(k)=\frac{\tilde{\Delta}^{3/2}}{\sin(\delta)\sqrt{v_F k}}\sin\left[\frac{\tan(\delta)}{2}\ln\left(\frac{v_F k}{\tilde{\Delta}}\right)+\delta\right], \tag{13}$$

where the phase δ is equal to $\arctan\sqrt{4C-1}$. In order to verify the boundary condition (12) the amplitude $\tilde{\Delta}(p=0)$ has to verify the following relation: $\tilde{\Delta}=(v_F\Lambda)\exp[-4\delta/\sqrt{4C-1}]$.

The solution of the electron self-energy has been derived for wave vectors larger than the cutoff $|\Delta_0|/v_F$ and shows to be independent of the amplitude $\Delta(r)=\Delta_0$ of the homogeneous Kekule distortion. The independence of Σ on the amplitude Δ_0 implies that the dynamical mass generation mechanism cannot be controlled by a homogeneous Kekule distortion.

It was shown elsewhere in the context of graphene bilayer that a gap opening in the energy spectrum can lead to an exciton instability under the specific amplitude of a short-ranged Coulomb interaction.[16] Here a similar excitonic instability could appear since the graphene monolayer spectrum sees a gap open directly induced by the Kekule distortion. Moreover the gap is enlarged by formation of a mass that is dynamically generated. An exciton instability could take place.[12] We address the question whether a Kekule distortion can amplify the mechanism of dynamical mass generation and consequently affect the exciton instability. The solution (13) for the electron self-energy Σ shows that the dynamically generated mass of the electron is independent of the homogeneous Kekule distortion. However the mass of the electron given by $m(k)=\sqrt{|\Delta_0|^2+\Sigma(k)^2}=\langle\bar{\Psi}(k)\Psi(k)\rangle$ shows that the exciton instability resulting from the electron-hole interaction is directly dependent on the homogeneous Kekule distortion but not through the mechanism of dynamical mass generation.

IV. CONCLUSIONS

The electron self-energy for graphene monolayer with a homogeneous Kekule distortion has been derived in the framework of quantum electrodynamics. It has been shown that the dynamical mass generation of the electrons resulting from the electron-hole interaction in the graphene monolayer is independent of the amplitude of the homogeneous Kekule distortion for a one-loop approximation. Such an independence of the dynamically generated mass provides an insight on the effects implied by lattice distortion on the energy spectrum of graphene monolayer.

The gap in the energy spectrum of the graphene monolayer is related to the mass of the electron, which is equal to $m(k)=\sqrt{|\Delta_0|^2+\Sigma(k)^2}=\langle\bar{\Psi}(k)\Psi(k)\rangle$. The Kekule distortion controls directly the gap by means of the amplitude $|\Delta_0|$ without any amplification through the dynamical mass Σ (due to the independence mentioned previously). Consequently exciton instability can be formed by direct relation to the homogeneous Kekule distortion and by the Kekule-independent mechanism of dynamical mass generation.

So far we tried to shed light on the effects induced by lattice distortions such as a homogeneous Kekule distortion $\Delta(r)=\Delta_0$. The present study does not admit any conclusion on the independence of the dynamical mass $m(k)$ in presence of an inhomogeneous Kekule distortion $\Delta(r) \neq \text{const}$. It is expected that inhomogeneous lattice distortion affects significantly the energy spectrum of graphene monolayer. As a consequence exciton instability could possibly be amplified through lattice distortion via nontrivial dynamical mass generation.

ACKNOWLEDGMENTS

The author is grateful to Jean Richert for having attracted his attention to articles about Kekule distortion and for enlightening comments on the present work. The author would like to thank Jung Hoon Han for enriching discussions.

APPENDIX: DERIVATION OF THE POLARIZATION FUNCTION

The polarization function $\Pi_{\mu\nu}$ is obtained by integrating the QED_3 action (5) over the fermionic field ψ and reads

$$\Pi^{\mu\nu}(q)=\frac{g^2}{\beta}\sum_{\sigma=\pm}\sum_{\omega_f}\int\frac{d^2\vec{k}}{(2\pi)^2}\mathrm{Tr}[G_0(k)\ \gamma^{\mu}\ G_0(k+q)\ \gamma^{\nu}],$$

where $G_0(k)=\frac{\gamma^{\rho}k_{\rho}+i\tilde{\tilde{\Delta}}_0}{k^2+|\Delta_0|^2}$ is the electron Green function and the trace operator Tr runs over the space of the gamma matrices. Focusing only on the static temporal component of the polarization $\Pi_{00}(q^0=0,\vec{q})$ and using the following relations $\mathrm{Tr}[i\tilde{\tilde{\Delta}}_0\gamma^0 i\tilde{\tilde{\Delta}}_0\gamma^0]=-4|\Delta_0|^2$ and $\mathrm{Tr}[\gamma^{\rho}k_{\rho}.\gamma^0\gamma^{\eta}(k_{\eta}+q_{\eta})\gamma^0]=4g^{(M)}_{\rho\eta}k_{\rho}(k_{\eta}+q_{\eta})$, where we defined the metric tensor $g^{(M)}_{\rho\eta}=\mathrm{diag}(1,-v_F^2,-v_F^2)$, the polarization function can be reduced to

$$\Pi^{00}(q)=\frac{g^2}{\beta}\sum_{\sigma=\pm}\sum_{\omega_f}\int\frac{d^2\vec{k}}{(2\pi)^2}\left(\frac{1}{k^2+|\Delta_0|^2}\times\frac{1}{(k+q)^2+|\Delta_0|^2}\right)\times\{4g^{(M)}_{\rho\eta}k_{\rho}(k_{\eta}+q_{\eta})-4|\Delta_0|^2\}. \tag{A1}$$

The computation of the polarization function can be simplified using the Feynmann identity $\frac{1}{ab}=\int_0^1 dx \frac{1}{(ax+(1-x)b)^2}$. Applying the change in variable $k \to k'-xq$ [in other words $\omega_f'=\frac{2\pi}{\beta}(n+1/2)$ and $\vec{k}'=\vec{k}+x\vec{q}$] and performing the sum over the fermionic Matsubara frequencies ω_f' one gets

$$\Pi^{00}(q^0=0,\vec{q})=\frac{\alpha}{\beta}\int \frac{d^2\vec{k}'}{(2\pi)^2}\int_0^1 dx\{S_1-2(v_F^2\vec{k}'^2+|\Delta_0|^2)S_2\}.$$

We define by S_1 and S_2 the sums over the Matsubara frequencies given by the following relations:[30]

$$S_1 \equiv \sum_{n=-\infty}^{\infty} \frac{1}{[\omega'^2+v_F^2\vec{k}'^2+x(1-x)v_F^2\vec{q}^2+|\Delta_0|^2]} = \frac{\beta^2}{4\pi Y}\tanh(\pi Y),$$

$$S_2 \equiv \sum_{n=-\infty}^{\infty} \frac{1}{[\omega'^2+v_F^2\vec{k}'^2+x(1-x)v_F^2\vec{q}^2+|\Delta_0|^2]} = -\frac{\beta^2}{8\pi^2}\frac{1}{Y}\frac{\partial S_1}{\partial Y},$$

where $\omega'=\frac{2\pi}{\beta}(n+1/2)$. The integration over the wave vector k' can be performed through the change in variable $Y=\frac{\beta}{2\pi}\sqrt{v_F^2\vec{k}'^2+x(1-x)v_F^2\vec{q}^2+|\Delta_0|^2}$ and finally the polarization function reads

$$\begin{aligned}\Pi^{00}(q^0=0,\vec{q}) &= \frac{\alpha}{\beta}\int_0^1 dx \lim_{\Lambda\to\infty}\int_{\Theta_q}^{\Lambda}\frac{2\pi}{(v_F\beta)^2}YdY\left[S_1+Y\frac{\partial S_1}{\partial Y}\right] \\ &\quad -\frac{\alpha}{\beta}\int_0^1 dx \lim_{\Lambda\to\infty}\int_{\Theta_q}^{\Lambda}\frac{dY}{2\pi}x(1-x)\vec{q}^2\frac{\partial S_1}{\partial Y} \\ &= \int_0^1 dx\left(\frac{\alpha}{2\pi v_F^2\beta}\right)\ln(2\cosh(\pi\Theta_q)) \\ &\quad -\left(\frac{\beta|\Delta_0|}{2}\right)^2\frac{\tanh(\pi\Theta_q)}{(\pi\Theta_q)},\end{aligned}$$

where $\Theta_q=(\frac{\beta}{2\pi})\sqrt{x(1-x)v_F^2\vec{q}^2+|\Delta_0|^2}$.

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